Conservation of Energy in Space

Recall that energy cannot be created or destroyed; it can only be converted from one form to another. In other words, regardless of what energy is being used or produced, the total energy contained in any system stays the same.

In space, any object:

- will always have gravitational potential energy relative to some galaxy, star, planet, or moon.
- may have kinetic energy, <u>if</u> it is moving relative to some galaxy, star, planet or moon.
- will not likely produce any significant heat or other form of wasted energy in its movement, due to the lack of friction from wind resistance.

Therefore, to calculate the total energy of a moving mass in space, we can state that:

$$\mathbf{E_t = E_k + E_p}$$

- substitute in the proper formulas to obtain

$$\mathbf{E_t = \frac{1}{2}mv^2 - \frac{GMm}{R}}$$

where:
→ **m** is the mass of the moving object
→**M** is the mass of the object exerting the gravitational pull (a planet, moon, etc.)
→**R** is the distance between the two objects
→**v** is the speed of the object

Use total energy to find unknown quantities for any mass that changes its speed or distance in space from a planetary or stellar object.

Example 12: A 12 000 kg spaceship is 7.2 x 10^8 m from the center of a planet that has a mass of 5.1 x 10^{25} kg. As it "falls" back to the planet's surface, the spaceship gains 9.0 x 10^{11} J of kinetic energy. What is the radius of the planet?

(see Gravitation Ex 12 for answer)

Conservation of energy can be used to determine a value called the *escape velocity* of a spacecraft. The escape velocity is the minimum velocity required for the craft to escape a planet's gravitational field. This means that:

- A body has "escaped" from a gravitational field if it is so far from the mass that generates the field that once it has *stopped*, its $\mathbf{E_p = 0}$.
- At a minimum speed, the spacecraft will (in theory) reach infinity and stop, so that $\mathbf{E_k = 0}$.

Therefore, in this system where the minimum velocity is required, the total energy $\mathbf{E_t = 0}$! In terms of conservation of energy:

total energy before = total energy after: $\mathbf{E_k + E_p = 0}$

Here the speed of the escaping object at the planet's surface must be great enough to provide the $\mathbf{E_k}$ needed so that $\mathbf{E_k = - E_p}$.

At the planet's surface, $\mathbf{E_p = -\dfrac{GMm}{R_p}}$ where $\mathbf{R_p}$ **= radius of the planet**

- $\therefore \quad \mathbf{\dfrac{GMm}{R_p} = \dfrac{1}{2}mv^2}$ ---> cancel spacecraft's mass and solve for **v**

Example 13: Determine the escape velocity for any spacecraft launched from the surface of Mars, which has a planetary mass of 6.40 x $\mathbf{10^{23}}$ kg and a radius of 3.4 x $\mathbf{10^{6}}$ m.

(see Gravitation Ex 13 for answer)

Finally, consider the total energy possessed by a satellite in a stable orbit.

Total energy = kinetic energy at *orbit speed* + potential energy at orbital altitude

Don't forget that **R = radius of planet + altitude**

To begin, the orbital speed must be determined. Since the satellite is orbiting in a circle, use $\mathbf{F_c = F_g}$ to find this speed.

Then, solve using $\mathbf{E_T = E_k + E_p}$ where $\mathbf{E_p = -\dfrac{GMm}{R}}$ (relative to infinity)

Example 14: Determine the total energy possessed by the Moon as it orbits the Earth.

(see Gravitation Ex 14 for answer)

Note that the total energy possessed by *any* object in a stable orbit is always equal to ***half*** of the potential energy contained in that object, or

$$\mathbf{E_{T\ (stable\ orbit)} = -\frac{1}{2}\frac{GMm}{R}}$$

This can be proven with algebraic substitution.